Dynamics and thermodynamics of automata with a visual cone. Comparison with a recursive thinking model

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Overview

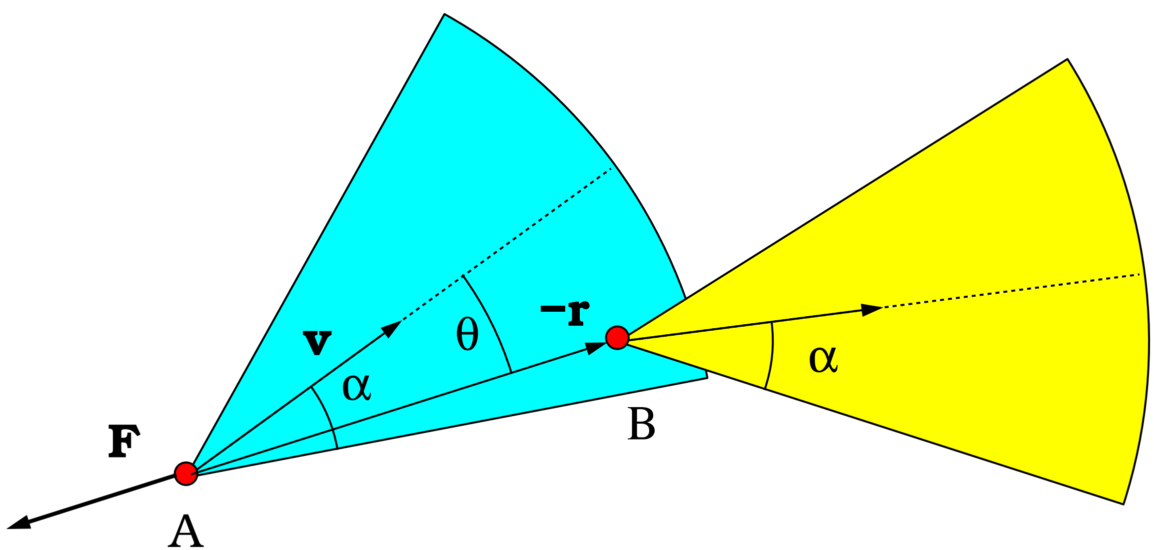
- We first discuss some results that we obtained modifying a known physical system to introduce a “zero level” approximation for a system of individuals provided with perception.
- Then we discuss possible applications to the problem of studying self organising patterns in the flow of pedestrians, comparing the results of models with different levels of “complexity”.

Gas of Automata

We modify a system of particles interacting with central forces introducing a cone of vision and thus non Newtonian effects due to perception. The forces are given by

$$\mathbf{F_{12}} = -\omega^2 \, \mathbf{r_1} + \frac{\mathbf{r_1} - \mathbf{r_2}}{r_{12}^2} \, \vartheta(C_1)$$

$$C_1 = (\mathbf{v_1} \cdot (\mathbf{r_2} - \mathbf{r_1}) - v_1 \, r_{12} \, \cos\alpha)\vartheta(r_v - r_{12})$$



Two automata problem

- Can be studied in the centre of mass (R, V) and relative (r, v) coordinates.
- In principle can be integrated but integration is cumbersome since R and r motion are not separated.
- Relative angular momentum $L = r \times v$ is conserved
- while relative energy switches according to cone conditions.

$$H_I = \frac{v^2}{2} + \frac{\omega^2 r^2}{2}$$

$$H_{II} = \frac{v^2}{2} + \frac{\omega^2 r^2}{2} + V(r)$$

$$H_{III} = \frac{v^2}{2} + \frac{\omega^2 r^2}{2} + V(r) + V(r)$$

One automaton in a force field

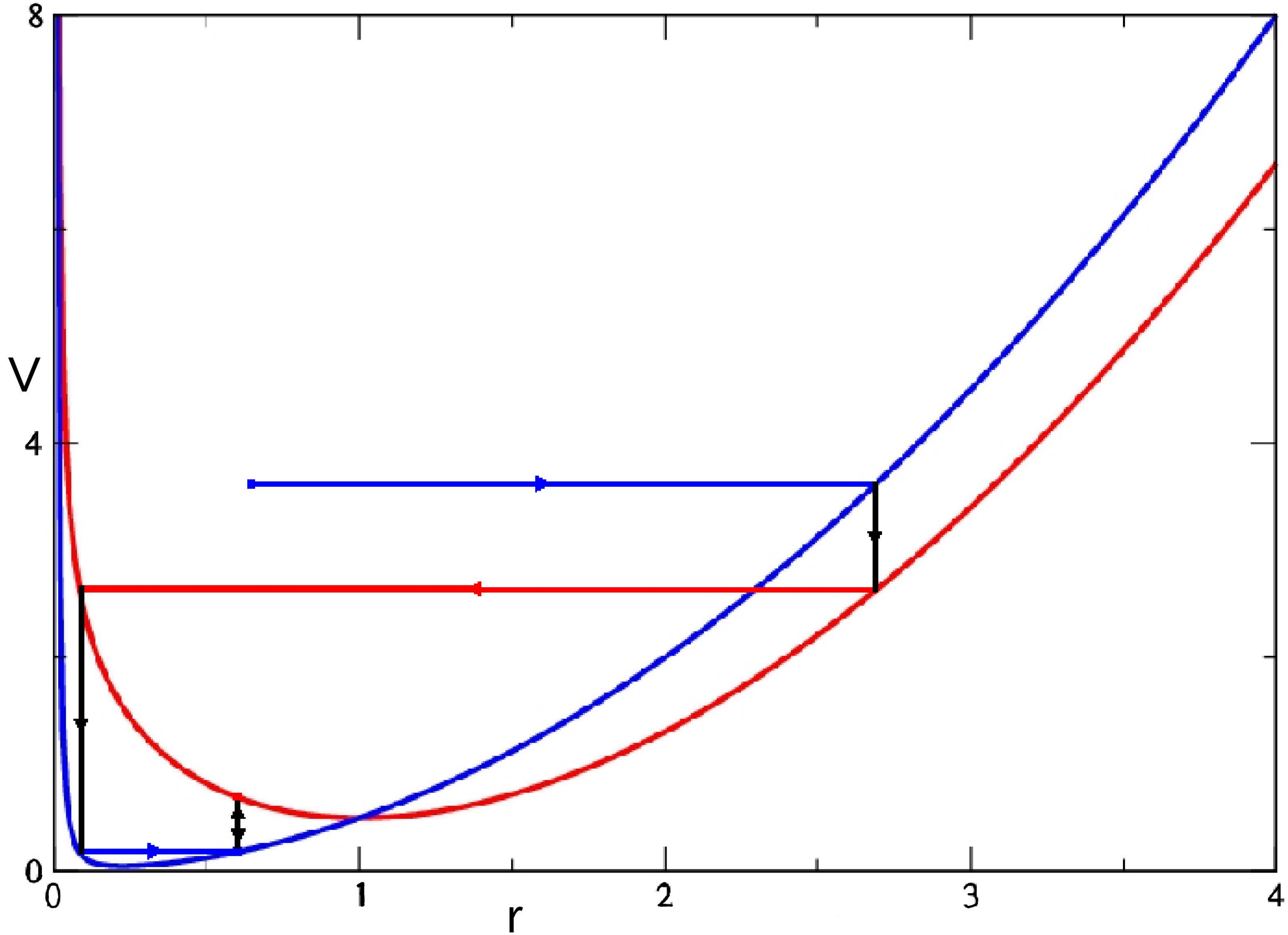
- Gives insight to the two body problem and can be easily analytically studied.
- It is described by a 1D Hamiltonian

$$H = \frac{\dot{r}^2}{2} + \frac{L^2}{2r^2} + \frac{\omega^2 r^2}{2} - \ln(r)\vartheta(C)$$

where $C = -r\dot{r} - L\cot\alpha$

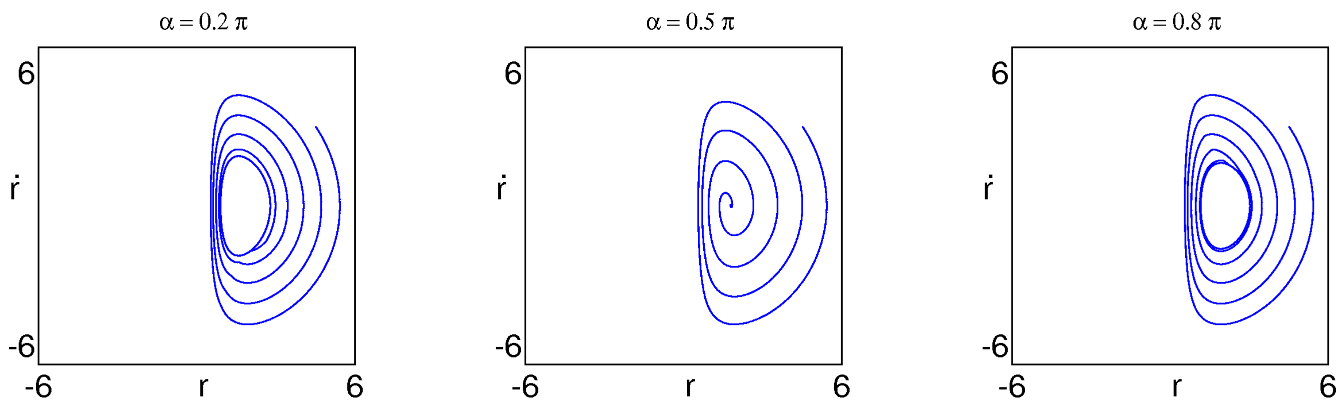
- This condition reduces to $\dot{r} < 0$ for $\alpha = \frac{\pi}{2}$.
- In this case switching and inversion points coincide and if located between the minima of potentials correspond to a stopping point (circular orbit).

Stopping condition



Numerical study of 1 and 2 automata

- The asymptotic orbits for the 1 and 2 automata problem with $\alpha < \pi$ are ellipses.



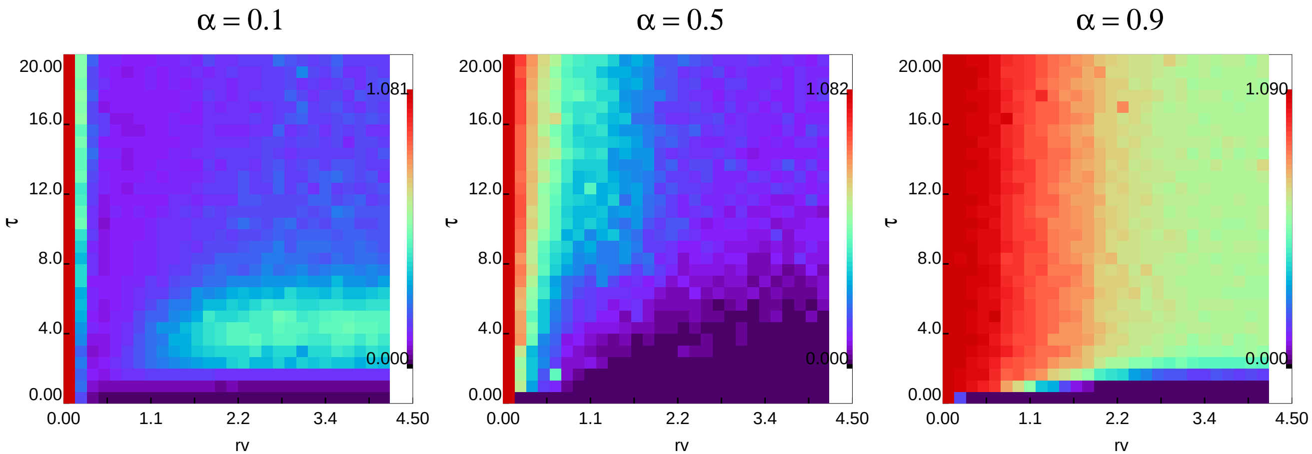
- The radius of the orbit for $\alpha = \frac{\pi}{2}$ can be predicted as the minimum of the potential corresponding to the asymmetric situation.

Energy dissipation

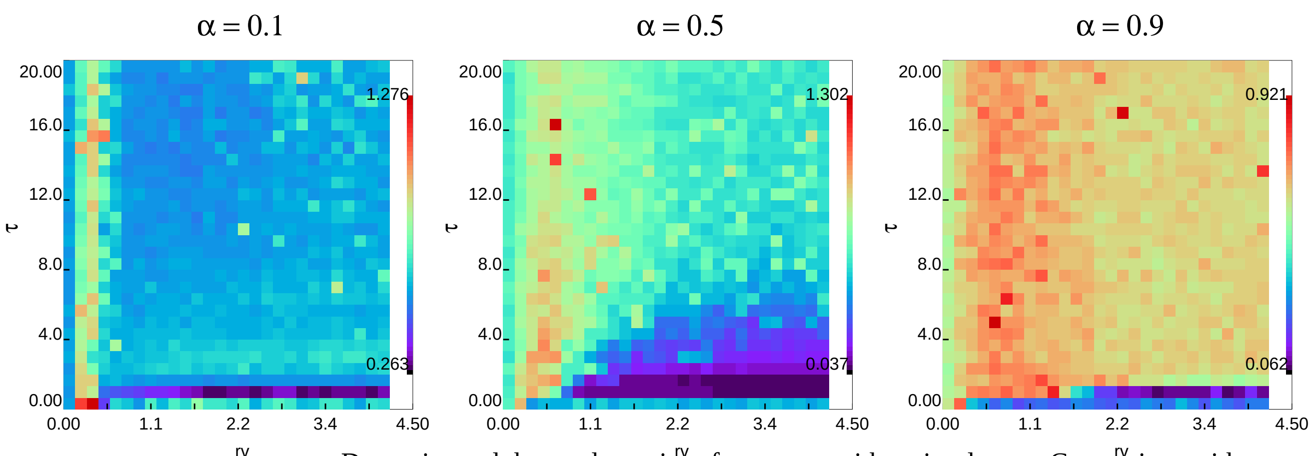
- Non Newtonian repulsive interaction as we introduced it causes dissipation.
- Our numerical study shows that for $N \geq 3$ the equilibrium state of the system has $T = 0$.
- And for $N \geq 4$ it does not present any ordered structure.
- Both $T > 0$ and ordered equilibrium states can be established introducing memory, i.e. allowing the automata to remember other automata for a time τ after they exit the cone.

Numerical analysis of equilibrium

Varying the control parameters α, r_v and τ we study in the equilibrium state the temperature

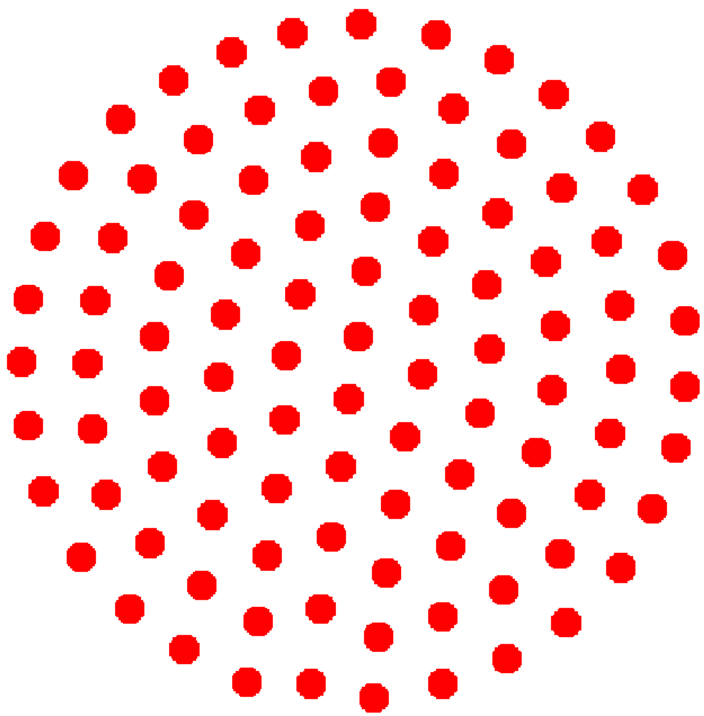


and the eventual formation of ordered structures.

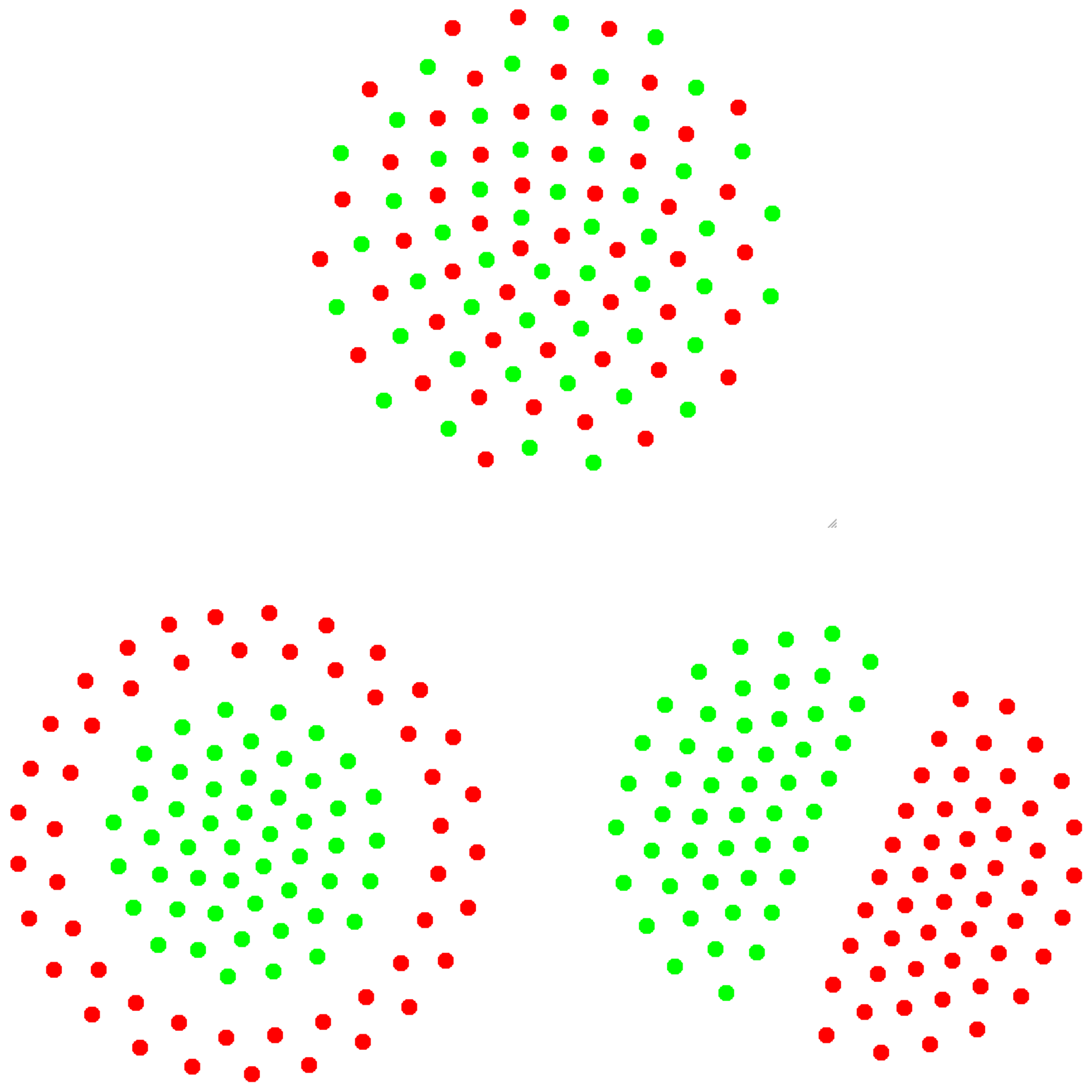


Crystals

The $T = 0$ equilibrium for low $\tau > 0$ and high enough r_v is an exagonal crystal



Crystals



Crowd dynamics patterns

- The patterns formed by crossing fluxes of pedestrians have been observed by Andō et al. (1988).
- At the crossing point stripes parallel to the bisector are formed.

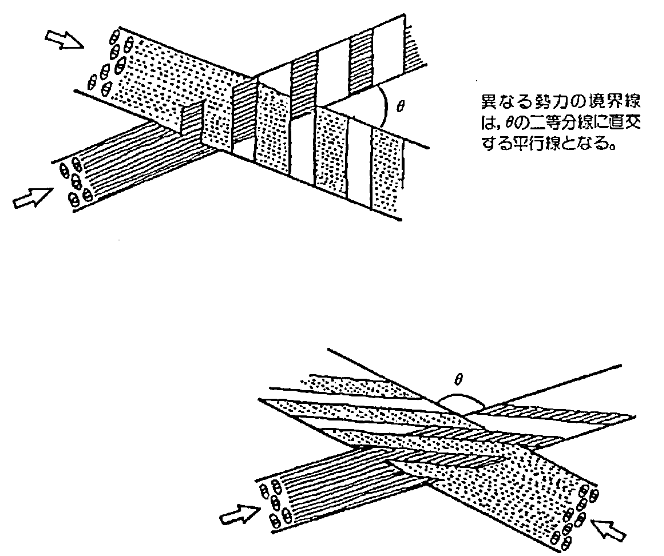
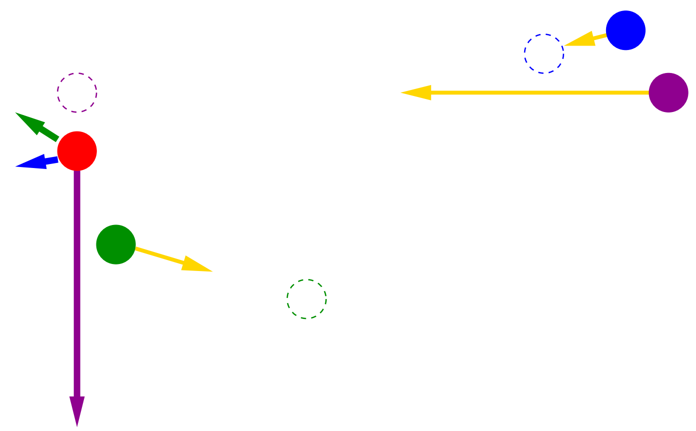


図3　交差流動の構造モデル

A collision avoiding model

- We have developed a collision avoiding model in which pedestrians are physically represented as hard discs whose dynamics is exactly solved
- At each time step each agent tries to predict the positions of all the agents falling in its cone of vision at time of a probable impact



- and applies a central impulsive force determined by these future positions to avoid the collision.

$$\mathbf{v}(t) = \mathbf{v}(t - \Delta_t) + \mathbf{f}(t)$$

$$\mathbf{x}(t + \Delta_t) = \mathbf{x}(t) + \mathbf{v}(t)$$

Evolution and results

- The details of the behaviour of each agent (including the possibility of “recursive thinking”) are coded in a genome
- and optimised with an evolutionary algorithm that uses a fitness function with a positive term due to velocity towards the goal and a negative one due to collisions (exchanged momentum or “pain”).

$$f = v_g - \beta \Delta p$$

- At the end of the process the desired result is obtained.
  angle between corridors $a = 0$
  angles between corridors $a = \frac{\pi}{8}$, $a = \frac{\pi}{4}$, $a = \frac{3\pi}{8}$
  angle between corridors $a = \frac{\pi}{2}$

Minimal requests for organisation

- These results show that a good collision avoiding model can reproduce the observed patterns.
- Nevertheless the presence of the patterns could not necessarily be related to a complex collision avoiding behaviour.
- We want thus to investigate if these patterns can be obtained on the base of pure physical interactions.

Driven discs on a torus

- Let us consider a channel with periodic boundaries on which discs undergoing elastic collisions (exactly solved) are subjected to a driving force to the goal and a frictional force

$$\dot{\mathbf{v}} = \mathbf{F_g} - \beta v$$

simulation discs

- and then charged discs that interact also through an electrostatic $\propto \frac{1}{r}$ force (also an interaction at distance with the walls is introduced).

simulation charged

simulation cone of vision

Non periodic conditions

- The use of periodic boundaries allows the system to reach equilibrium in a arbitrary long time and minimises problems due to the stability of patterns.
- The ability to self organise using non periodic conditions (and higher densities) can be attained letting discs to interact only with those that have a different goal. simulation goal dependent
- The differences with the behaviour of actual pedestrians can be noticed when using crossing fluxes simulation goal dependent cross
- and when studying more complex problems as bottlenecks or doors.

Future development

- We want to perform a quantitative numerical and to some extent analytical study on the formation of self organised patterns in a channel (critical densities, number of lanes under different conditions, time to reach equilibrium) comparing purely physical systems with systems using “cognitive” strategies of different levels of complexity.
- The comparison with more complex topologies will help us in understanding which kinds of patterns are sure signals of the presence of some kind of cognitive strategy.
- This work is part of a research project that studies, with methods coming both from theoretical physics and computer science, the dynamics of complex systems composed by a large number of individuals with cognitive properties, with a particular attention to the dynamics of “mobility systems”.
- http://www.physycom.unibo.it   http://www.fisicadellacitta.it