Dynamics and thermodynamics of a gas of automata

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We propose the gas of Von Neumann automata as a model for complex systems, such as a crowd or a swarm [1]. Our system is similar to a gas of charged particles confined by an external potential, except for the perception, described by a visual cone of range R and angular half-aperture α. An automaton A is repelled by an automaton B, when it falls within its visual cone, and feels no interaction when B is out of the cone. The repulsion decreases with the distance and we assume an inverse proportionality as in the 2D electrostatic case. More explicitly the force is given by

$$\mathbf{F} = \begin{cases} \frac{\mathbf{r}}{r^2} & \text{if } B \in C_A \\ 0 & \text{if } B \notin C_A \end{cases}$$

where $\mathbf{r} = \mathbf{r_A} - \mathbf{r_B}$ is the displacement and $r = |\mathbf{r}|$ is the distance of A from B. The cone condition is defined by

$$B \in C_A \quad \text{if} \quad r < R \quad \text{and if} \quad |\theta| < \alpha \qquad \cos\theta = -\frac{\mathbf{r}\cdot\mathbf{v}}{r\,v}$$

where $\mathbf{v}$ is the velocity of A, along which we chose the axis of the cone. The

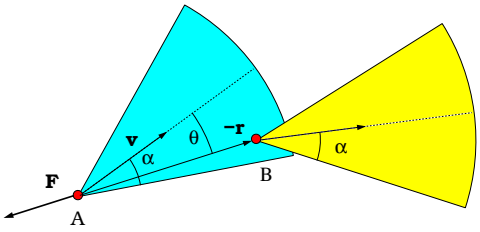

Figure 1: Automaton B falls in the cone of vision of automaton A, which feels a repulsive force $\mathbf{F}$, while B doesn't feel any force caused by the presence of A

model is supposed to describe a low density crowd, even though in this case the interactions are certainly more involved: social attractive forces [2], responsible of the formation of small clusters, are certainly present in addition to misanthropy. The presence of the visual cone renders the force non Newtonian and changes significantly the N automata problem with respect to the N body problem, by introducing a sort of damping. The two automata

problem can be solved, by reducing the relative motion to the dynamics of a particle moving in a central time dependent force field. Since the angular momentum L is preserved we consider the radial motion, whose effective potential changes with time, according to the cone condition. The automaton reaches a relative equilibrium whenever its inversion point is between the minima of the effective potentials acting when the cone condition is verified or when it is not. The asymptotic orbits of the center of mass and of the relative motion are closed curves, with an energy lower than the initial one. For the N automata problem a numerical investigation shows that the average kinetic energy decreases producing a sort of "freezing" of the system. The minimum is reached when $\alpha = \frac{\pi}{2}$. To counteract this freezing process we have introduced the memory. The automaton A feels a repulsion from B for a time interval τ after it escapes from its visual cone. By letting $\tau \to \infty$ we recover the case of N interacting charges without any visual cone, where the total energy and the average kinetic energy are preserved. We have investigated how the system, subject to en external harmonic potential, reaches an asymptotic quasi equilibrium thermodynamic state, depending on the control parameters α, R, τ. This is a very naive model of people moving in an open space with a single attracting point, which renders the system symmetric by rotation. Another simpler case is the avenue, where we consider two groups moving in opposite directions and study the emergence of organized patterns [3]. In presence of a constant force field and a dissipation the two groups reach a constant opposite velocity, in the absence of mutual interactions. The repulsive mutual interaction causes self organization phenomena with the formation of streams, whose properties depend on the vision parameters. The model might be used to simulate the approach to the equilibrium of a system of individuals with contrasting goals, after introducing some heterogeneity by sampling the vision parameters and the mutual force strength.

References

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